Emittance In An Electron Storage Ring

In an electron storage ring, with an isomagnetic guide field, the horizontal emittance of the electron beam is given by[Sa1]

$$\varepsilon = C_q \frac{\gamma^2}{\rho J_x} \frac{1}{2\pi\rho} \int_{\text{dipoles}} H\,ds = C_q \gamma^2 \frac{I_5}{I_2 - I_4} \qquad (1)$$

where $H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$,

$$J_x(\delta) = 1 - \frac{1}{2\pi\rho^2} \int_{\text{dipoles}} \eta_x (\delta - 2n) ds \qquad (2)$$

$$\text{with } \delta = \begin{cases} 1 & \text{for a sec tor magnet} \\ 0 & \text{for a parallel magnet} \end{cases} \qquad (3)$$

and

$$n \equiv -\frac{\rho}{B}\frac{dB}{dx} \qquad \& \qquad C_q \equiv \frac{55\hbar}{32\sqrt{3}mc} = 3.84 \times 10^{-13} \text{ m} - \text{rad}. \qquad (4)$$

In general a light source lattice is constructed from ‘basic cells’. A cell contains dipoles, quadrupoles and sextupoles and is usually bounded by dispersion free drift spaces for insertion devices.

To explore the differences between the various types of lattices it is useful to rewrite (1) as

$$\varepsilon = F(\nu_x, \text{lattice}) \frac{E^2[\text{GeV}]}{J_x N_d^3} \quad \text{m} - \text{rad}, \qquad (5)$$

where N_d is the number of dipole magnets, E is the energy in GeV.

For a fixed N_d & E the main variation in (5) is due to F(ν_x , lattice), since $0 < J_x < 3$ and is typically $J_x \approx$ 1-2. The following table lists the approximate minimum values of F (ν_x) for several types of lattices. A realistic lattice will have a slightly higher value than the theoretical minimums given here. The table also mentions an example of each lattice type to give the reader an idea of the basic cell and the betatron and dispersion functions. The emittances for these example rings are not necessarily given by the minimum value.

Lattice Type	F_{min}	N_d/cell	**Example**
FODO*	7.28×10^{-4}	2	SPEAR
DBA (Chasman-Green)	2.36×10^{-5}	2	NSLS VUV
Isomag. Matched TBA**	1.56×10^{-5}	3	BESSY II
Theor. Minimum Emit.*	7.84×10^{-6}	1	

* dispersion suppressers must be added to yield $\eta = 0$ insertions
** for the Isomagnetic Matched TBA the center dipole is $3^{1/3}$ times longer than the end dipoles (Le2)

References: He2, Ka1, Le2, Ri1, So1, Te1, Wi1

Beam Size and Angular Divergence

The horizontal and vertical beam sizes are given by,

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \sigma_\varepsilon^2 \eta_x^2} \qquad \& \qquad \sigma_y = \sqrt{\varepsilon_y \beta_y}\,.$$

The horizontal and vertical angular divergences are given by,

$$\sigma'_x = \sqrt{\varepsilon_x \gamma_x + \sigma_\varepsilon^2 \eta_x'^2} \qquad \& \qquad \sigma'_y = \sqrt{\varepsilon_y \gamma_y}\,,$$

where $\gamma_i = \frac{1+\alpha_i^2}{\beta_i}$, $\alpha_i = -\frac{\beta'_i}{2}$, $\varepsilon_y = \frac{\chi\varepsilon}{1+\chi}$, $\varepsilon_x = \frac{\varepsilon}{1+\chi}$ and χ is defined to be the emittance coupling.