Electron Beam Optics and Twiss Parameters[Br1]

The particle position and angular deviation are given by,

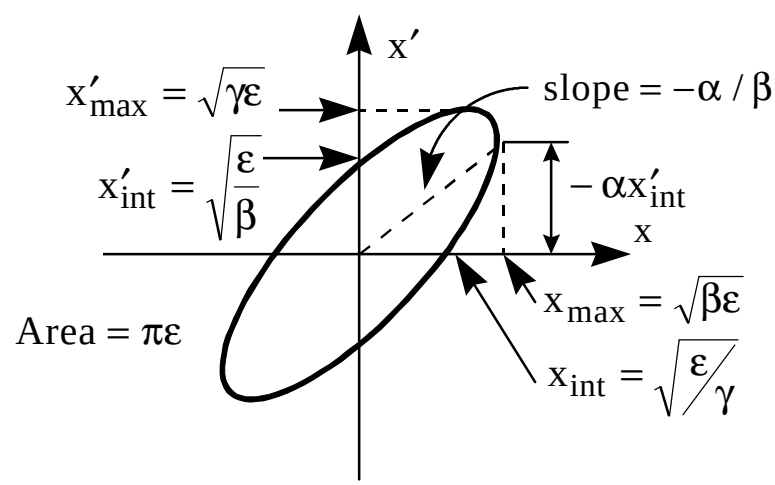
$$x(s) = \sqrt{\varepsilon\beta(s)}\cos[\psi(s)+\phi]$$

$$x'(s) = -\sqrt{\frac{\varepsilon}{\beta(s)}}\left[\alpha(s)\cos[\psi(s)+\phi]+\sin[\psi(s)+\phi]\right]$$

$$= \sqrt{\varepsilon\gamma(s)}\cos[\chi(s)+\phi]$$

where $\psi(s) \equiv \int_0^s \frac{d\tau}{\beta(\tau)}$, $\alpha(s) \equiv -\frac{\beta'(s)}{2}$, $\gamma(s) \equiv \frac{1+\alpha^2(s)}{\beta(s)}$ and $\chi(s)$ satisfies $\tan[\psi(s)-\chi(s)] = 1/\alpha(s)$. The trio ($\alpha$, β, γ) are referred to as the Twiss parameters. The invariant emittance ellipse is given by

$$\gamma(s)x^2 + 2\alpha(s)xx' + \beta(s)x'^2 = \varepsilon\,.$$



To transform the ellipse to a circle the following normalized variables are useful,

$$X \equiv \frac{x}{\sqrt{\beta}} \qquad \& \qquad X' \equiv \alpha\frac{x}{\sqrt{\beta}} + \sqrt{\beta}x'\,.$$